A summary of the minor research project done by Mary Elizabeth Antony,

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Introduction of different topological concepts in the theory of frames

With the work of Marshall Stone on the topological representation of Boolean algebras and distributive lattices, the connection between topology and lattice theory began to be explored.

As a dual notion of category of frames we have the category of locales. The category of pointless topological space called locales is an extension of ordinary topological space .

In locales the concept of points gets replaced with "opens". Thus the introduction to the concepts of pointless topology was laid. Most concepts of point set topology like separation axioms ,compactification ,etc… have already be defined into the contexts of locales and proved the analogues theorems.

In this project we have defined second countablity and separability in the broader context of frames.

The following are the summary of the work done.

1 L-base

Definition : A collection B_L of elements of a locale L is said to be L-base for the locale L if for every a(≠0) ∈ L ,there exists a non empty sub-collection $\{b_i : b_i \in B, i \in \Delta\}$ such that

1.1Example $\bigvee_{i\in\Delta} b_i \leq a$ and $\bigvee\{b_i : b_i \in B\} = 1$.

In the background of frames the class B_L of all singletons form a base for Discrete topology on X

1.2 Remark 1

If B_L is a L-base for the frame (locale) L iff ∀ p∈ L there exists some $b_p \in B_L$ such that $b_p \leq p$.

1.3 Remark 2

Let B_L be a L base for some frame(locale) L B_{L}* be a class containing B_L ie,

$B_L \subseteq B_L$* then B_L * is also a L -base for L.

1.4 Theorem

Let B_L be a collection of elements of L .Then B_L is a L-base for L if for any b_1,,$b_2 \in B_L$ there exists some $b_3 \in B_L$ such that $b_3 \leq b_1 \wedge b_2$..

Proof

Let B_L be a L-base then for $b_1, b_2 \in B_L$, $b_1 \wedge b_2 \in$ L (L being a frame).Then ,by the definition of base there exists $b_3 \in B_L$ such that $b_3 \leq b_1 \wedge b_2$.

2 Lower frame

2.1Definition :Let L be a frame with base B_L and let L* be a frame with base B_{L^*} * Then we say that L is a lower frame to L*,denoted as L $\prec B$ L* if for every b$\in B_L$

b =$\vee$\{b*: b* $\in B_{L^*}$*\}.

2.2 Example : R with usual topology is a lower frame to R with upper limit topology.

3 2-countability

3.1 Definiton

A frame L is said to be a $B_L{}^2$ frame if it has a countable L- base. And this property is called 2-countability.

3.2 Example

The frame of Upperclosed sets of finite list of bits 0and 1 is second countable.

3.3 Theorem

2-countability is hereditary.

4 Lindeloff

If every cover U = {U_α: α∈ A} of a frame L has a countable sub cover then L is said to be Lindeloff.

4.1 Theorem

Every 2- countable frame L is Lindeloff.

Proof

Let B_L be a countable L- base for L .Suppose U is any cover of L .For each U∈ U there is some $b_{1,U}$ ∈ B_L such that $b_{1,U} \leq U$.

Now B''={ $b_{1,U}$:U∈ U } is a countable set , since $B'' \subseteq B_L$, say B'' = {$b_{i,U1}$, $b_{2,U2}$, $b_{3,U3}$,..................}

SinceB_L is a L base we know that 1 =⋁{b:b∈ B_L} .Then U_1,U_2,..... is a countable sub-cover from *U.*

4.2 Remark

If L is $B_L{}^2$ then clearly the set of all minimal elements is countable.

5 . Separability

As in classical topology, a frame is said to be separable if it has a countable dense sublocale.

5.1 Theroem

Every $B_L{}^2$ frame is separable.

5.2 Lemma

Let h:A→B be a frame homomorphism. If a is a regular element of A, then h(a) is a regular element in B

5.3 Theorem

Homomorphic image of a separable space is separable

Proof

Let A be separable and let h:A→B is a frame homomorphism.

Since A is s separable , A┐┐ is a countable dense sublocale of A. Now consider h(A┐┐).Clearly this is a countable and dense. Hence h(a) is separable.

6. Conclusion

Frame theory finds its applications in computer science where the domain theory provides a mathematical foundation for semantics of programming languages. Here we have developed the concepts of second countability and separability in locale theory. Further study can be done in developing an equivalent theory for local base and first countability